

# DEEP BED FILTRATION: MODELLING THEORY AND PRACTICE

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## Contents

1. Introduction
  - 1.1. Conditions of Deep Bed Filtration
  - 1.2. Stages of Deep Bed Filtration
  - 1.3. Transport Mechanisms
    - 1.3.1. Interception
    - 1.3.2. Inertial Impaction
    - 1.3.3. Sedimentation
    - 1.3.4. Diffusion
    - 1.3.5. Hydrodynamic Action
    - 1.3.6. Straining
  - 1.4. Attachment Mechanisms
    - 1.4.1. Long-Range Forces
    - 1.4.2. Short-Range Forces
    - 1.4.3. Presence of Primary and Secondary Minima
    - 1.4.4. Effect of Surface Blocking and Particle Size
  - 1.5. Detachment Mechanisms
  - 1.6. Modeling Approaches
    - 1.6.1. Microscopic Modeling
    - 1.6.2. Macroscopic Modeling
2. Mathematical Models for the Initial Filtration Stage
  - 2.1. Removal Efficiency
  - 2.2. Model Representation of Porous Media
  - 2.3. Classification of Microscopic Models
  - 2.4. Eulerian Methods (Convective Diffusion Analysis)
    - 2.4.1. Perfect-Sink Models
    - 2.4.2. Non-Penetration Models
  - 2.5. Lagrangian Methods (Trajectory Analysis)
  - 2.6. Semi-Empirical Formulations
    - 2.6.1. Correlations for the Collision Efficiency  $\alpha$
    - 2.6.2. Correlations for the Initial Collection Efficiency  $\eta_0$
  - 2.7. Limitations of Initial Stage Models
3. Mathematical Models for the Transient Filtration Stage
  - 3.1. Macroscopic Models

- 3.1.1. Methods of Solution to Macroscopic Equations
- 3.2. Transient Stage Models
  - 3.2.1. Retained Particle Detachment Models
  - 3.2.2. Deposition Site Saturation Models
  - 3.2.3. Particle Blocking Models
  - 3.2.4. Surface Charge Models
- 3.3. Limitations of Macroscopic and Transient Stage Models
- 3.4. The Effect of Particle Size Distribution
- 4. Conclusion
- Glossary
- Bibliography
- Biographical Sketches

## Summary

Deep bed filtration is a rapid and efficient method for removing small particles from liquids. Such dispersions of particles in liquids are common in a wide range of industries; of particular interest to the civil or environmental engineer are those found in water and wastewater treatment. The removal of particles by deep bed filtration involves a complex array of mechanisms, which can be broadly classified as transport mechanisms, attachment mechanisms, and detachment mechanisms. This article provides an overview of the conditions and stages under which deep bed filtration occurs, and presents a summary of the mechanisms which govern the filtration process.

## 1. Introduction

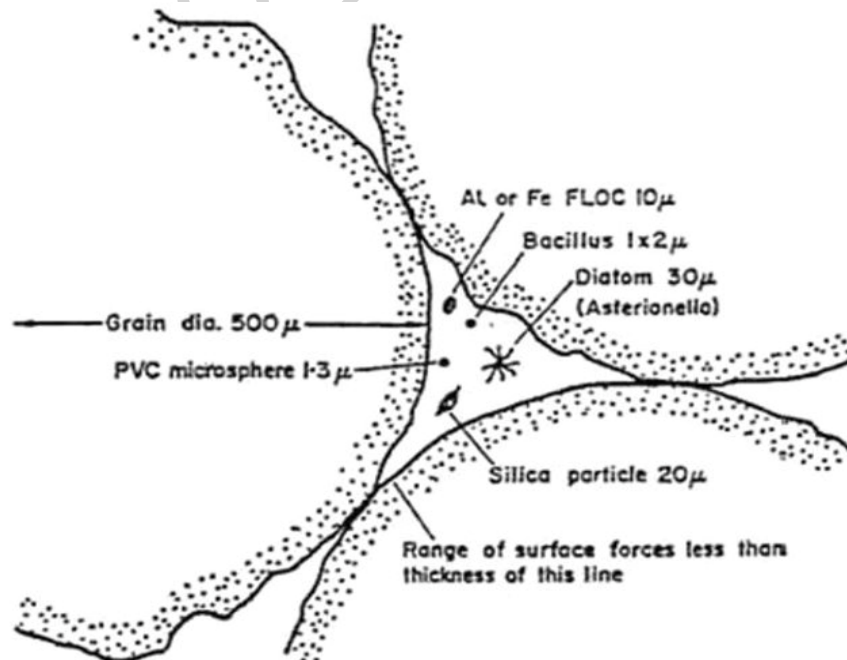


Figure 1. Typical dimensions of particles and filter grains in aqueous deep-bed filtration (Ives, K. J. (1970) Rapid filtration. *Water Research*, 4, 201-223).

In deep bed filtration, particles are removed when passed through beds of granular or fibrous filter material. Deep bed filtration differs from other kinds of filtration in that the solid particles suspended in the fluid are generally smaller than the pores of the filter medium, as shown in Figure 1.

As the suspension travels through the filter, the particles deposit at differing depths on the filter grains which constitute the bed. In engineering practice, deep bed filtration usually is used to treat raw water after the processes of coagulation, flocculation and sedimentation.

### **1.1. Conditions of Deep Bed Filtration**

The surface charge of suspended particles and filter grains is important in deep-bed filtration. So-called favorable conditions occur when particles and filter grains are of opposite charges so that the particles and filter grains undergo an attractive interaction.

Conversely, under unfavorable conditions repulsive interactions between particles and filter grains of like charge occur. In water or wastewater treatment, conditions are predominantly unfavorable due to the negative surface charge of both the filter media and the suspended particles.

### **1.2. Stages of Deep Bed Filtration**

The filtration process consists of two stages: the initial stage, and the transient stage. In the initial stage, particle deposition occurs onto a clean filter; that is, particles are directly deposited onto filter grains. This particle deposition has a negligible effect on the properties of the filter during the initial stage.

The transient stage occurs after the initial stage, and describes the remainder of the filtration process. Deposition occurs on filter grains which are already partially covered by deposited particles.

During this stage, the performance characteristics of the filter change dramatically: the particle removal can increase or decrease depending on the factors such as available surface area, collection by deposited particles or changes in interstitial (through pores) velocity. This means the performance of the filter changes substantially over the duration of a filter run.

The transient stage is often divided into three sub-stages; these are the ripening stage, working stage and breakthrough stage, as shown in Figure 2. The ripening stage is characterized by the improvement in filter effluent quality over time, or the decrease in the ratio of effluent concentration  $C$  to influent concentration  $C_0$ . The working stage, in which particle removal remains essentially constant, generally follows the ripening stage. Finally, the break-through stage generally occurs in the latter part of filtration, where the filter efficiency decreases with time. It has been observed however that ripening and breakthrough can occur simultaneously in suspensions of varying particle size, as some particle sizes can show improvement while others deteriorate.

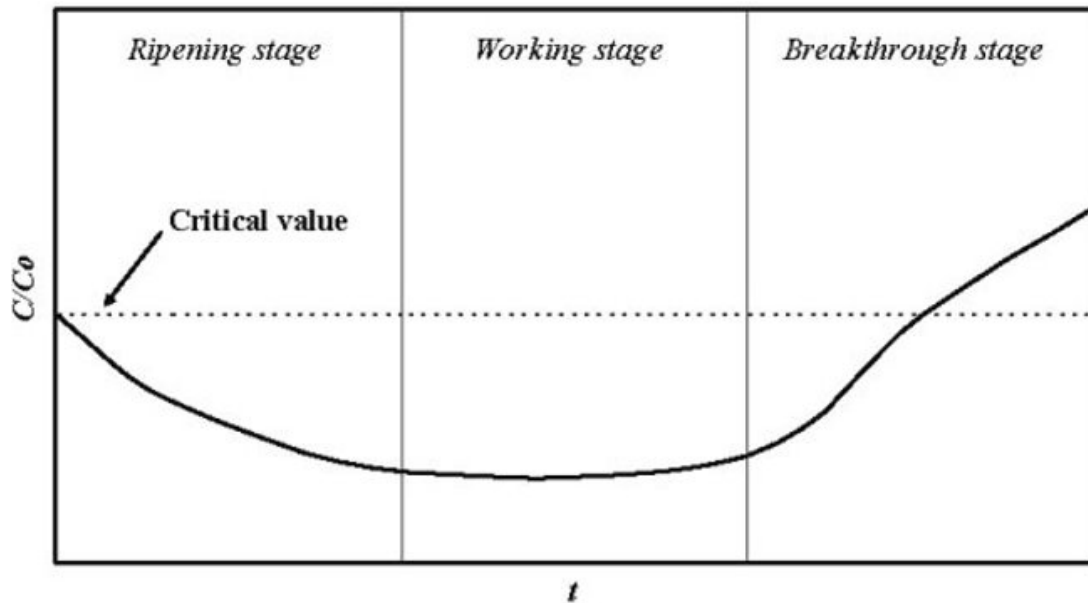


Figure 2. Typical curve showing filter performance over different stages of filtration.

### 1.3. Transport Mechanisms

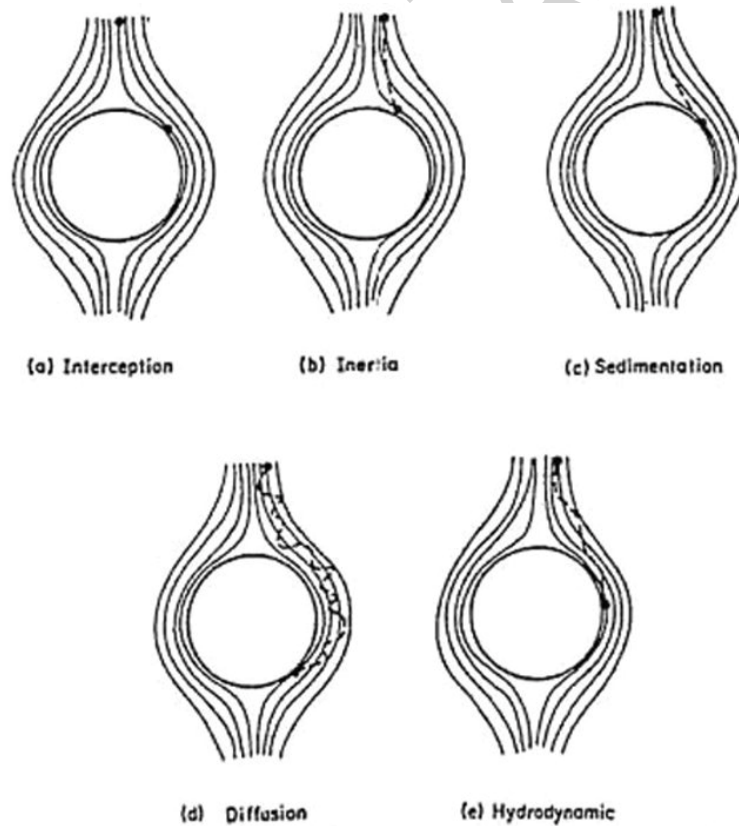


Figure 3. Schematic representation of the common transport mechanisms in deep-bed filtration (Ives, K. J. (1970) Rapid filtration. *Water Research*, 4, 201-223).

Before particles can be captured by filter grains, they must first be transported sufficiently near the grain surface so that surface forces can bind the particle to the grain. A range of transport mechanisms have been observed, and are outlined in the following, with schematic representations of the main mechanisms shown in Figure 3. It should be emphasized that these various mechanisms do not act in isolation; suspended particles will bear their effects to varying degrees depending on flow conditions, filter geometry, and the physicochemical properties of the particles.

### 1.3.1. Interception

The transport of particles by interception occurs due to the finite size of particles. Consider a particle in a fluid stream; if all forces on the particle are negligible, the particle shall follow the fluid streamlines. Interception occurs when the streamlines approach the filter grains to within a particle radius. The mechanism of interception is characterized by the dimensionless number  $N_R$ , which is the ratio of particle diameter  $d_p$  to grain diameter  $d_c$ .

### 1.3.2. Inertial Impaction

As fluid streamlines approach a filter grain, they must diverge as the flow passes around the grain. However, particles entrained in these streamlines, due to their inertia, do not change trajectories as the streamlines do. If the particle has sufficient inertia, it may follow a trajectory that will lead to collision with the filter grain surface. This inertial impaction is characterized by the dimensionless Stokes number, given by:

$$N_{st} = \frac{\rho_p d_p^2 U}{18\mu d_c} \quad (1)$$

where  $\rho_p$  is the density of the particle,  $U$  is the fluid approach velocity, and  $\mu$  is the dynamic viscosity of the fluid.

### 1.3.3. Sedimentation

If the density of a particle is greater than that of the fluid it is suspended in, the particle will settle out of suspension in the direction of gravity. This mechanism is characterized by the gravitational parameter  $N_G$ , given by:

$$N_G = \frac{(\rho_p - \rho) d_p^2 g}{18\mu U} \quad (2)$$

where  $\rho$  is the fluid density and  $g$  is the acceleration due to gravity. This can be seen to be the ratio of the particle Stokes velocity to the fluid approach velocity  $U$ .

### 1.3.4. Diffusion

For very small particles, a random movement is observed due to the thermal vibrations

of the fluid molecules; this is termed Brownian diffusion. For particles of diameter  $> 1 \mu\text{m}$ , these effects are minimal due to viscous drag forces and torques exerted by the fluid. However, for particles of diameter  $< 1 \mu\text{m}$  (submicron or Brownian particles), this effect becomes important, and increases with decreasing particle size. The mechanism of diffusion is characterized by the Peclet number, given by:

$$N_{Pe} = \frac{d_c U}{D} \quad (3)$$

where  $D$  is the Stokes-Einstein diffusion coefficient. The Peclet number represents the relative importance of diffusion and convection in a fluid flow; i.e. a low Peclet number characterizes a flow dominated by diffusive effects, and a high Peclet number characterizes a flow dominated by convective effects.

### 1.3.5. Hydrodynamic Action

In deep-bed filtration, flow through the filter bed is generally laminar with some velocity profile, and hence a shear field. For a uniform shear field and a spherical particle, rotation of the particle causes a swerving path across the shear field. For non-uniform shear field and aspherical particles (as is usually the case in practice), the particle is deflected in a similar, but unpredictable manner. The overall effect, termed hydrodynamic action, is of an apparently random drifting across the flow streamlines which may lead to contact with the filter grain surface. Efforts to characterize the importance of this effect via a suitable dimensionless number have so far been unsuccessful, due largely to the geometrical complexity of a typical filter pore arrangement.

### 1.3.6. Straining

When particles in suspension are larger than the pores in the filter medium, they will be retained by the process commonly known as straining. However, the presence of significant straining is undesirable in deep-bed filtration, as a filter cake or mat forms, clogging the bed rapidly. This requires a considerable increase in the pressure gradient required across the filter bed to maintain the desired flow rate, and does not efficiently make use of the entire depth of the filter. However, straining may often be unavoidable in practical situations, particularly for polydisperse solutions which may contain large particles.

## 1.4. Attachment Mechanisms

After transport to the filter grain surface, particles are then attached to the surface of the grain by a variety of mechanisms. These mechanisms depend greatly on the chemical characteristics of the filter system, and are generally considered to be substantially more important in determining filter performance than the initial transport stages. The forces which govern the attachment of particles to the filter grain surfaces can be broadly divided into long-range forces and short-range forces, as discussed below.

### 1.4.1. Long-Range Forces

Long-range forces refer to those surface forces which influence particle transport and attachment at distances of up to approximately 100 nm. The primary long-range forces are listed as follows.

#### London – van der Waals Force

The London – van der Waals (VDW) force refers to the attraction between two closely separated surfaces, and is caused by momentary pairwise oscillations in electron density, causing the creation of instantaneous dipole moments that draw the interacting particles together. This force is mostly responsible for particle collection and adhesion; without it, hydrodynamic effects would prevent particles from reaching filter grain surfaces.

There are a multitude of expressions for the VDW force for different geometries; in deep-bed filtration, the interaction is usually assumed to occur between a sphere (the particle) and a flat plate (the filter grain, as it is many times the size of the particle and appears locally to be flat).

Broadly speaking, there are two approaches towards calculation of this force: these are Hamaker’s microscopic approach, in which the interaction is calculated by considering a pairwise summation of all intermolecular interactions; and the macroscopic approach of Lifshitz theory, which relies entirely on macroscopic electrodynamic properties of the two surfaces.

While the Lifshitz theory is more rigorous, its use has been limited by lack of electrodynamic data for particles and filter media, and complexity in formulation for practical geometries.

Thus, the Hamaker approach is almost universally used, with various corrections given to account for retardation of the VDW force (due to the finite time of propagation between bodies).

Results are normally given in terms of the interaction energy per unit area,  $V_A$ , and are tabulated below in Table 1, where  $H$  is the Hamaker constant,  $h$  is the separation distance between bodies,  $a_p$  is the particle radius,  $h^*$  is the dimensionless separation distance  $h/a_p$ , and  $\lambda'$  is a characteristic interaction wavelength assumed to be 100 nm. The VDW force  $F_A$  can then be calculated by the following expression:

$$F_A = \frac{-\partial V_A}{\partial h} \quad (4)$$

Expression		Notes	Source
$V_A = -H / 6h^*$	(1)	Unretarded	Hamaker

			(1937)
$V_A = \frac{-H}{6h^*} \left[ \frac{1}{1+14h/\lambda'} \right]$	(2)	Retarded; valid for separations up to approximately 20% of particle diameter.	Gregory (1981)
$V_A = H[2.45\lambda'/(60\pi)\{(h-a_p)/h^2 - (h+3a_p)/(h+2a_p)^2\} - 2.17\lambda'^2/(720\pi^2)\{(h-2a_p)/h^3 - (h+4a_p)/(h+2a_p)^3\} + 0.59\lambda'^3/(5040\pi^3)\{(h-3a_p)/h^4 - (h+5a_p)/(h+2a_p)^4\}]$	(3)	Retarded; valid for separations greater than approximately 8 nm.	Czarnecki (1979)

Table 1. Approximations for London-van der Waals interaction energy derived by various researchers.

### Electrical Double-Layer (EDL) Interactions

Most surfaces in an aqueous environment have surface charges, which are balanced by an equivalent amount of counter-ions present in the solution. This double layer of charge is characterized by the zeta potential, which is the electrical potential between the bulk of solution and the outer region of the double layer. The classic treatment of the interaction between two double layers was developed by Derjaguin and Landau and Verwey and Overbeek, and is known as the DLVO theory. A variety of expressions for the double-layer force  $F_{DL}$  are given in the literature; possibly the most widely used is the analytical expression of Hogg et al., given by:

$$F_{DL} = \frac{\hat{\epsilon} a_p (\zeta_p^2 + \zeta_c^2) \kappa \exp(-\kappa h)}{2(1 - \exp(-2\kappa h))} \left[ 2 \frac{\zeta_p \zeta_c}{\zeta_p^2 + \zeta_c^2} - \exp(-\kappa h) \right] \mathbf{n} \quad (5)$$

where  $\hat{\epsilon}$  is the dielectric constant of the liquid,  $\zeta_p$  and  $\zeta_c$  are the zeta potentials (an approximation of the surface potential) of the particle and collector (filter grain) respectively,  $\mathbf{n}$  is the unit normal vector, and  $\kappa$  is the inverse Debye length, given by:

$$\kappa = \sqrt{\frac{4\pi e^2}{\hat{\epsilon} k T} \sum_j z_j^2 m_j} \quad (6)$$

where  $e$  is the charge of an electron,  $k$  is the Boltzmann constant,  $T$  is the absolute temperature, and  $m_j$  is the concentration of the  $j^{\text{th}}$  ion species present in the solution with valence  $z_j$ .



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### Biographical Sketches

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